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MIXED PROBLEMS FOR A STRIP PARTIALLY COUPLED TO A RIGID BASE*

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Dynamic and static problems are considered for an elastic strip partially coupled along its lower face to a nondeformable base; shearing displacements are specified along its upper face. Mixed boundary-value problems are then reduced, by means of an integral Fourier transformation, to an integral equation of the first kind for the tangential contact stresses along the coupling segment. The solution of the integral equation is reduced to the solution of an infinite system of linear algebraic and of special type equations. Certain results from a numerical solution of these problems are presented.

1. Let us consider a mixed static problem for an elastic strip h units high, adhering along its lower face to an absolutely rigid base along the segement Ω and lying without any friction outside Ω. Shearing displacements are specified along the upper face of the strip, and there are no normal stresses present. The boundary conditions of this problem have the form (problem A)

$$u(x, h) = U_0(x), \quad \sigma_y(x, h) = 0, \quad v(x, 0) = 0, \quad |x| < \infty$$
$$u(x, 0) = 0, \quad x \in \Omega; \quad \tau_{xy}(x, 0) = 0, \quad x \in \bar{\Omega}$$

In addition to problem A, we consider an analogous dynamic problem for an elastic strip in the case of steady-state oscillation conditions (problem B). The boundary conditions for this problem have the form

$$u(x, h, t) = U_0(x) e^{-i\omega t}, \quad \sigma_y(x, h, t) = 0, \quad v(x, 0, t) = 0$$
$$|x| < \infty$$
$$u(x, 0, t) = 0, \quad x \in \Omega; \quad \tau_{xy}(x, 0, t) = 0, \quad x \in \bar{\Omega}$$

Below we will consider the following variants of the segment Ω:

$[-a; +a]$ (problems A1, B1)
$[-b; -a] \cup [+a; +b]$ (problems A2, B2)
$(-\infty; -a] \cup [+a; +\infty)$ (problems A3, B3)

These mixed boundary-value problems may be reduced, by means of an integral Fourier transformation, to an integral equation of the form

$$\int_\Omega k_1(x-\xi) T(\xi) d\xi = \Delta \int_{-\infty}^{\infty} k_2(x-\xi) U_0(\xi) d\xi, \quad x \in \Omega \tag{1.1}$$

$$k_j(t) = \int_\Gamma K_j(u) e^{iut} du, \quad K_j(u) = \frac{M_j(u)}{N(u)}, \quad j = 1, 2 \tag{1.2}$$

$$M_1(u) = 2u + (3-4\nu) \operatorname{sh} 2u, \quad M_2(u) = 2(1-\nu) u \operatorname{ch} u - u^2 \operatorname{sh} u$$
$$N(u) = u \operatorname{ch}^2 u, \quad \Delta - 4\mu h^{-1} \quad \text{(problem A)}$$
$$M_1(u) = \sigma_1\sigma_2 \operatorname{sh} \sigma_2 \operatorname{ch} \sigma_1 = u^2 \operatorname{sh}\sigma_1 \operatorname{ch} \sigma_2$$
$$M_2(u) = \sigma_1 [(u^2 - {}^1/_2 \varkappa_2^2) \operatorname{ch} \sigma_1 - u^2 \operatorname{ch} \sigma_2]$$
$$N(u) = \sigma_1 \operatorname{ch} \sigma_1 \operatorname{ch} \sigma_2, \quad \sigma_j = \sqrt{u^2 - \varkappa_j^2}, \quad j = 1, 2$$
$$\varkappa_1^2 = \rho\omega^2 h^2/(\lambda + 2\mu), \quad \varkappa_2^2 = \rho\omega^2 h^2/\mu, \quad \Delta = 2\mu h^{-1} \quad \text{(problem B)}$$

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Here $T(\xi)$ are the tangential stresses along the segment Ω for problem A or their amplitude values for problem B; ρ, ν, λ, and μ respectively are the density, Poisson coefficient, and Lamé coefficients of the material of the strip, and ω is the angular frequency.

The functions $K_j(u)$ are real on the real axes and meromorphic in the complex plane. Moreover,

$$K_1(u) \sim |u|^{-1} + O(e^{-2|u|}),\ u \to \infty \quad \text{(problem A)} \tag{1.3}$$
$$K_1(u) \sim |u|^{-1} + O(|u|^{-3}),\ u \to \infty \quad \text{(problem B)}$$
$$K_2(u) \sim e^{-|u|} + O(e^{-3|u|}),\ u \to \infty \quad \text{(problems A, B)}$$

In view of (1.3), we have for $K_1(u)$

$$K_1(u) = K_1(0) \prod_{n=1}^{\infty} (u^2 - \zeta_n^2)(u^2 - z_n^2)^{-1} \tag{1.4}$$

where ζ_n and z_n are the zeroes and poles of the function $K_1(u)$ in the complex plane $u = \alpha + i\beta$, which grow in modulus with increasing number, thereby ensuring the convergence of the infinite product (1.4).

The poles of problem A are of two-stages and in the complex plane, i.e., $z_{2m-1} = z_{2m}$. In the case of the dynamic problem B, a finite number of zeroes and poles may lie on the real axis, depending upon the frequency ω. The integration path Γ from problem B is selected in accordance with the radiation conditions as in /1/. In problem A, the path Γ coincides with the real axis. Note that $K_1(u)$ may be readily factored (thanks to the representation (1.4))

$$K_1(u) = K_{1+}(u) K_{1-}(u), \quad K_{1+}(u) = \sqrt{K_1(0)} \prod_{n=1}^{\infty} (u - \zeta_n)(u - z_n)^{-1} \tag{1.5}$$

where ζ_n and z_n are the zeroes and poles from the upper half-plane.

2. Let us pass on to the solution of the integral equation (1.1). Supposing that $U_0(x)$ may be expanded in a Fourier series, we construct a solution for a special form of the right side of Equation (1.1) $U_0(x) = e^{i\varepsilon x}$. In this case, the integral equation (1.1) may be written in the form

$$\int_{\Omega} k_1(x - \xi) T(\xi) d\xi = 2\pi \Delta K_2(\varepsilon) e^{i\varepsilon x}, \quad x \in \Omega \tag{2.1}$$

Because of the properties of the kernel $K_1(u)$ from (1.3) and (1.4), the solution of equation (2.1) will be found in the form of a series in eigenfunctions of the integral operator /2/. For problem A1,

$$T(x) = A_0 e^{i\varepsilon x} + \sum_{n=1}^{\infty} (A_n e^{i\zeta_n(a+x)} + C_n e^{i\zeta_n(a-x)}) \tag{2.2}$$

Here and below, A_0, A_n, and C_n are given constants to be determined and ζ_n are the zeroes of the function $K_1(u)$ lying in the upper half-plane.

The solution of problem B1 will be found in the same form as problem A1. It must be borne in mind that solutions of the problems in the form (2.2) have been previously found /2/.

To determine the unknown constants in the solution of equation (2.1), we satisfy it by direct substitution (2.2). Since taken account of two-stages of the poles $K_1(u)$ for problem A1, once the quadratures have been computed, we obtain from the theory of residues an infinite system of linear algebraic equations for determining the sets of unknowns A_n and C_n. This system is transformed and, passing to the unknowns $x^{\pm}_n = A_n \pm C_n$, we rewrite it in matrix form:

$$AX^{\pm} = \mp BX^{\pm} + D^{\pm} \tag{2.3}$$
$$X^{\pm} = \{x_n^{\pm}\},\ A = \{a_{mn}\},\ B = \{b_{mn}\},\ D^{\pm} = \{d_m^{\pm}\} \tag{2.4}$$
$$a_{2m-1,n} = (z_{2m-1} - \zeta_n)^{-1}, \quad a_{2m,n} = (z_{2m}' - \zeta_n)^{-2}$$
$$b_{2m-1,n} = (z_{2m-1} - \zeta_n)^{-1} e^{2ia\zeta_n}, \quad b_{2m,n} = (z_{2m} - \zeta_n)^{-2} e^{2ia\zeta_n}$$
$$d_{2m-1}^{\pm} = -A_0 [(z_{2m-1} - \varepsilon)^{-1} e^{-i\varepsilon a} \pm (z_{2m-1} + \varepsilon)^{-1} e^{i\varepsilon a}]$$
$$d_{2m}^{\pm} = -A_0 [(z_{2m} - \varepsilon)^{-2} e^{-i\varepsilon a} \pm (z_{2m} + \varepsilon)^{-2} e^{i\varepsilon a}]$$
$$A_0 = \Delta K_1(\varepsilon)/K_2(\varepsilon); \quad m = 1, 2, \ldots; \quad n = 1, 2, \ldots$$

The infinite system of linear algebraic equations for problem B1 obtained by direct substitution of (2.2) in (2.1) has the same form as in /2/, and may be written in matrix form (2.3) with the following notation

$$a_{mn} = (z_m - \zeta_n)^{-1}, \quad b_{mn} = (z_m + \zeta_n)^{-1} e^{2ia\zeta_n} \qquad (2.5)$$
$$d_m^{\pm} = -A_0 [(z_m - \varepsilon)^{-1} e^{-ia\varepsilon} \pm (z_m + \varepsilon)^{-1} e^{ia\varepsilon}]$$

In the case of problems A2 and B2, the integral equation (2.1) may be reduced to two integral equations /3/:

$$\int_a^b [k_1(x-\xi) + k_1(x+\xi)] T_+(\xi)\, d\xi = 2\pi\Delta \operatorname{ch}(i\varepsilon x) K_2(\varepsilon) \qquad (2.6)$$
$$\int_a^b [k_1(x-\xi) - k_1(x+\xi)] T_-(\xi)\, d\xi = 2\pi\Delta \operatorname{sh}(i\varepsilon x) K_2(\varepsilon)$$
$$T_{\pm}(x) = {}^1/_2 [T(x) \pm T(-x)]$$

The solution of equations (2.6) is found for problems A2 and B2 in the form

$$T_+(x) = A_0 \operatorname{ch}(i\varepsilon x) + \sum_{n=1,3,5}^{\infty} (A_n e^{i\zeta_n(|x|-a)} + C_n e^{i\zeta_n(|x|-b)}) \qquad (2.7)$$
$$T_-(x) = A_0 \operatorname{sh}(i\varepsilon x) + \operatorname{sign}(x) \sum_{n=2,4,6}^{\infty} (A_n e^{i\zeta_n(|x|-a)} + C_n e^{i\zeta_n(|x|-b)})$$

where A_0, A_n, and C_n are unknown constants found from the integral equation (2.6). For example, for problem B2 is the infinite system of linear algebraic equations obtained by direct substitution of (2.7) in (2.6) may be written in matrix form as follows:

$$AX^{\pm} = \mp BX^{\pm} \pm C(X^+ - X^-) \pm D(X^+ + X^-) + F^{\pm} \qquad (2.8)$$
$$X^{\pm} = \{x_n^{\pm}\}, \quad A = \{a_{mn}\}, \quad B = \{b_{m,n}\}, \quad C = \{c_{mn}\}$$
$$D = \{d_{mn}\}, \quad F^{\pm} = \{f_m^{\pm}\}, \quad x_n^{\pm} = A_n \pm C_n$$
$$a_{mn} = (z_m - \zeta_n)^{-1}, \quad b_{mn} = {}^1/_2 (z_m + \zeta_n)^{-1} e^{i\zeta_n(b-a)}$$
$$c_{mn} = {}^1/_2 (z_m + \zeta_n)^{-1} e^{2iaz_m}$$
$$d_{mn} = {}^1/_2 (z_m - \zeta_n)^{-1} \exp[2iaz_m + i\zeta_n(b-a)]$$
$$f_m^{\pm} = -(A_0/2)[(z_m + \varepsilon)^{-1} e^{ib\varepsilon} - (z_m - \varepsilon)^{-1} e^{-ib\varepsilon}] \mp$$
$$(A_0/2)\{(z_m - \varepsilon)^{-1} e^{ia\varepsilon} \pm (z_m + \varepsilon)^{-1} e^{-ia\varepsilon} \mp$$
$$e^{2iaz_m}[(z_m + \varepsilon)^{-1} e^{ia\varepsilon} \pm (z_m - \varepsilon)^{-1} e^{-ia\varepsilon}]\}$$
$$A_0 = \Delta K_2(\varepsilon)/K_1(\varepsilon); \quad m = 1, 2, \ldots; \; n = 1, 2, \ldots$$

The integral equation (2.1) for problems A3 and B3 may also be reduced to two integral equations analogous to (2.6) with the only difference that the limits of integration are now from a to ∞. The solution of problems A3 and B3 must be found in the form

$$T_+(x) = A_0^+ \operatorname{ch}(i\varepsilon x) + \sum_{n=1}^{\infty} A_n^+ e^{i\zeta_n(|x|-a)} \qquad (2.9)$$
$$T_-(x) = A_0^- \operatorname{sh}(i\varepsilon x) + \operatorname{sign}(x) \sum_{n=1}^{\infty} A_n^- e^{i\zeta_n(|x|-a)}$$

The infinite system of linear algebraic equations for finding the sets of unknown constants $A_n^{\pm}$ has, for instance, the following form for problem B3:

$$AX^{\pm} = \pm BX^{\pm} + D^{\pm}, \quad X^{\pm} = \{A_n^{\pm}\} \qquad (2.10)$$
$$a_{mn} = (Z_m - \zeta_n)^{-1}, \quad b_{mn} = (z_m + \zeta_n)^{-1} e^{2iaz_m}$$
$$d_m^{\pm} = -\frac{A_0^{\pm}}{2}\left[\frac{e^{i\varepsilon a}}{z_m - \varepsilon} \pm \frac{e^{-i\varepsilon a}}{z_m + \varepsilon} \mp e^{2iaz_m}\left(\frac{e^{i\varepsilon a}}{z_m + \varepsilon} \pm \frac{e^{-i\varepsilon a}}{z_m - \varepsilon}\right)\right]$$
$$A_0^{\pm} = \Delta K_2(\varepsilon)/K_1(\varepsilon), \quad m = 1, 2, \ldots; \; n = 1, 2, \ldots$$

3. Above it was shown that all our mixed problems may be reduced to the solution of integral equations of the first kind of the form (2.1), (2.6). In turn, the integral equations of mixed problems reduce to the solution of infinite systems of a special type of linear algebraic equations, in each of which the column of unknown constants is multiplied by a

singular matrix. To solve these systems, it is necessary to regularize them, i.e., reduce them to an algebraic system of the second kind $X = GX + H$, after which a decision is made as to whether they must be regularized. Regularization of this special type of infinite systems reduce to the inversion of the singular matrix A. Inversion of the singular matrix of the form (2.5), (2.8), (2.10) has been previously presented in /2/; here it was a matter of inverting of Wiener-Hopf integral operator with corresponding kernel. A formula has also been given in this paper according to which it is possible to compute the elements of the matrix A^{-1} inverse to A:

$$A^{-1} = \{\tau_{nk}\}, \quad \tau_{nk} = [K_+'(-\zeta_n) \quad [R_-^{-1}(z_k)]'(z_k - \zeta_n)]^{-1} \tag{3.1}$$

For purposes of completeness of our study, we construct a matrix inverse to the singular matrix of the form (2.4). For this purpose, it is first necessary to find the solution of the Wiener-Hopf integral equation

$$\int_0^\infty \varphi(x) k(x-\xi) d\xi = 2\pi f(x), \quad 0 < x < \infty; \quad k(t) = \int_\Gamma K_1(u) e^{iut} du \tag{3.2}$$

where $K_1(u)$ is given by the formulas (1.2). In view of the properties of $K_1(u)$, the right side is selected in the special form

$$f(x) = e^{iz_k x} + ixe^{iz_k x} \tag{3.3}$$

The solution of the integral equation (3.2) with right side (3.3) is found by the Wiener-Hopf method and has the form

$$\varphi(x) = \frac{1}{K_+(z_k)} \sum_{n=1}^{\infty} \left[\frac{1}{z_k - \zeta_n} - \frac{1}{(z_k - \zeta_n)^2}\right] \frac{e^{i\zeta_n x}}{K_+'(-\zeta_n)} \tag{3.4}$$

Here and in (3.3), ζ_n and z_n are the zeroes and poles of $K_1(u)$ in the upper half-plane of the complex plane $u = \alpha + i\beta$. On the other hand, the solution of (3.2) may be found in the form

$$\varphi(x) = \sum_{n=1}^{\infty} x_n e^{i\zeta_n x} \tag{3.5}$$

and then (3.5) directly substituted in (3.2). As a result, we determine x_n by finding the infinite system

$$\sum_{n=1}^{\infty} x_n(z_k) \lim_{\gamma \to z_{2r-1}} \frac{d}{d\gamma}\left[\frac{(\gamma - z_r)^2}{(\gamma - \zeta_n) K^{-1}(\gamma)}\right] = \delta_{kr} \tag{3.6}$$
$$\sum_{n=1}^{\infty} x_n(z_k) \frac{2}{(z_{2r} - \zeta_n)[K^{-1}(z_{2r})]''} = \delta_{kr}$$

But from (3.4), $x_n(z_k)$ have the form

$$x_n(z_k) = \frac{1}{K_+(z_k) K_+'(-\zeta_n)}\left[\frac{1}{z_k - \zeta_n} - \frac{1}{(z_k - \zeta_n)^2}\right]$$

Performing some computations with (3.6) and bearing in mind the final relation, we obtain

$$\sum_{n=1}^{\infty} x_n(z_k) P_k \left(\frac{1}{z_{2r-1} - \zeta_n}\right) = \delta_{kr}$$
$$\sum_{n=1}^{\infty} x_n(z_k) q_k \left(\frac{1}{z_{2r} - \zeta_n}\right)^2 = \delta_{kr}, \quad q_k = -\frac{P_k^2}{P_k - S_k + Q_k}$$
$$P_k = \frac{2M_1(z_k)}{N''(z_k)}, \quad S_k = \frac{2M_1'(z_k)}{N''(z_k)}, \quad Q_k = \frac{2N'''(z_k) M_1(z_k)}{3(N''(z_k))^2}$$

Since $z_{2k-1} = z_{2k}$, the elements of the inverse matrix $A^{-1} = \{\tau_{nk}\}$ may be written in expanded form as follows:

$$\tau_{n,2k-1} = \frac{2M_1(z_{2k-1})}{K_+(z_k) K_+'(-\zeta_n) N''(z_{2k-1})}\left(\frac{1}{z_{2k-1} - \zeta_n} - \frac{1}{(z_{2k-1} - \zeta_n)^2}\right) \tag{3.7}$$
$$\tau_{n,2k} = -\frac{6M_1^2(z_{2k})}{3[M_1(z_{2k}) - M_1'(z_{2k})] N''^2(z_{2k}) + M_1(z_{2k}) N'''(z_{2k})} \times$$

$$\frac{1}{K_+(z_{2k})\,K_+'(z_{2k})}\left(\frac{1}{z_{2k}-\zeta_n}-\frac{1}{(z_{2k}-\zeta_n)^2}\right)$$

Thus, once we have the formulas for the inverse matrices (3.1) and (3.7), all our infinite systems (2.4), (2.5), (2.8), and (2.10) may be regularized by multiplication of A^{-1} on the left (the existence of a left-hand inverse matrix has been previously proved /2/). As a result we obtain regularized infinite systems of the second kind, and in case (2.8), a system of two regularized infinite systems

$$X^{\pm} = \mp A^{-1}BX^{\pm} \pm A^{-1}C\,(X^+ - X^-) \pm A^{-1}D\,(X^+ + X^-) + A^{-1}F^{\pm}$$

As was done in /2/, it is possible to prove that infinite regularized systems of linear algebraic equations will be quasi-completely regularized /4/.

4. As an example, let us consider the case $\varepsilon = 0$, i.e., the case of constant displacements on the upper face $U_0(x) \equiv 1$.

In order to create a numerical realization of problems A and B, it is first necessary to find the zeroes and poles of the kernel $K_1(u)$ of the integral equation (2.1). The poles of problem A may be found analytically: $z_n = i(\pi/2 + \pi n)$, and the first several zeroes with $\nu = 0.3$ and $\nu = 0.2$ are as follows:

$\pm 0.8032 + 2.1650i$; $\pm 1.2509 + 5.3821i$; $\pm 1.4785 + 8.5533i$; $\pm 1.6340 + 11.7115i$;. . . $(\nu = 0.3)$;
$\pm 0.6889 + 2.1842i$; $\pm 1.1478 + 5.3908i$; $\pm 1.3769 + 8.5590i$; $\pm 1.5326 + 11.7156i$; . . . $(\nu = 0{,}2)$

The zeroes and poles of problem B depend both on ν and on the generalized frequency $\varkappa_2$. Several values of ζ_n and z_n obtained in a computation with $\nu = 0.3$ are given below:

$\zeta_n = 2.350$; $0{,}192$; $\pm 1{,}321 + 4.747i$; $\pm 1.515 + 8.165i$; $\pm 1.657 + 11{,}430i$;
$z_n = 2{,}788$; $0{,}677$; $3{,}459i$; $4.391i$; $7.173i$; $7.666i$; $10.520i$; $10{,}862i$; . . . $(\varkappa_2 = 3.2)$;
$\zeta_n = 4{,}628$; 2.088; $\pm 1.248 + 3{,}616i$; $\pm 1.514 + 7.562i$; $\pm 1.662 + 11.007i$; $z_n = 4.747$; $2{,}162$; 1.671; $3.881i$; $6.057i$; $7.385i$; $9.793i$; $10.666i$; . . . $(\varkappa_2 = 5.0)$.

In this example, the infinite systems for problems A and B are significantly simplified. After this simplification, the systems must be regularized, as shown in Sect.3, as a result of which the systems become quasi-completely regular; the method of reduction may then by applied to solve them on the computer. The arrangement of the system that would ensure a desired precision for the solution of the integral equation (2.1) increases with increasing value of the parameter $\lambda = h/a$; in the case of problem B, the dimension increases with increasing generalized frequency $\varkappa_2$.

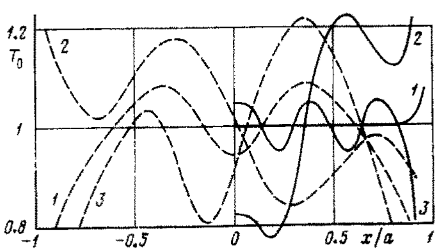

Fig.1

To find the contact stresses with a 0.5% error, it was established computationally that if $\lambda < 2$ and $\varkappa_2 < 6$, in the worst case we may limit ourselves to an algebraic system with dimension $N = 40$.

A program package in the Unified System FORTRAN OS ES language was compiled for numerical realization of problems A and B. With the ES-1022 computer, the maximal machine time for problem A amounted to 8 minutes, and for problem B, 25 minutes.

In Fig.1, the solid curves depict the distribution of the dimensionless contact stresses $T_0(x)$, where $T_0(x)\Delta\varkappa_2/\sin\varkappa_2 = \mathrm{Re}\,[T(x)]$, depending on the generalized frequency in the case Ω: $[-a; +a]$ with $\lambda = 0.25$. Curve 1 corresponds to the case $\varkappa_2 = 0$ (problem A1), and curves 2 and 3 to cases $\varkappa_2 = 4$ and $\varkappa_2 = 5$ (problem B1). If $\varkappa_2 \ll 1$, the numerical values of the solution of problem B virtually coincide with the solution of problem A.

The broken lines given the distribution of the dimensionless contact stresses along a coupling segment Ω of length $2a$, assuming this coupling segment is unique (curve 1) or if there exists a second coupling segement on the left of the same length and at a distance $4a$ units from the first segment (curve 2) or at a distance a units from the first segment (case 3). The computations were conducted for $\lambda = 0.5$ and $\varkappa_2 = 5$.

Displacements of the surface of pole outside Ω may be found from the formula

$$u(x, 0, t) = \frac{e^{-i\omega t}}{2\pi}\left[\frac{1}{\Delta}\int_{\Omega} k_1(x-\xi)\,T(\xi)\,d\xi - \int_{-\infty}^{\infty} k_2(x-\xi)\,U_0(\xi)\,d\xi\right], \quad x \in \bar{\Omega}$$

For example, in the case of problem B1 we have

$$u(x, 0, t) = \frac{e^{-i\omega t}}{\cos \varkappa_2} + \sum_{m=1}^{\infty} A_m^* e^{i[z_m(x-a)-\omega t]}$$

$$A_m^* = \frac{M_1(z_m)}{N'(z_m)} \left[\frac{A_0}{\varepsilon - z_m} (1 - e^{2iaz_m}) + \sum_{n=1}^{\infty} \frac{A_n}{\zeta_n - z_m} \times \right.$$

$$\left. (e^{2ia\zeta_n} - e^{2iaz_m}) - \sum_{n=1}^{\infty} \frac{C_n}{\zeta_n + z_m} (1 - e^{2ia(\zeta_n + z_m)}) \right]$$

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